

① $\mathcal{L} = \frac{1}{16\pi G} R \sqrt{-g} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \sqrt{-g} - V(\phi) \sqrt{-g}$

* $\phi = \bar{\phi}(t) + \phi(t, \vec{x})$

* $g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + h_{\mu\nu}(t, \vec{x})$

② Background

* $ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \Rightarrow H(t) \equiv \dot{a}/a \approx \dot{q}/q(t) \equiv -\frac{\dot{a}}{a^2}$

* Einstein $\Rightarrow \left\{ \begin{array}{l} 3H^2 = 8\pi G [\frac{1}{2} \dot{\phi}^2 + \bar{V}] \\ (-1+2q)H^2 = 8\pi G [\frac{1}{2} \dot{\phi}^2 - \bar{V}] \end{array} \right\}$

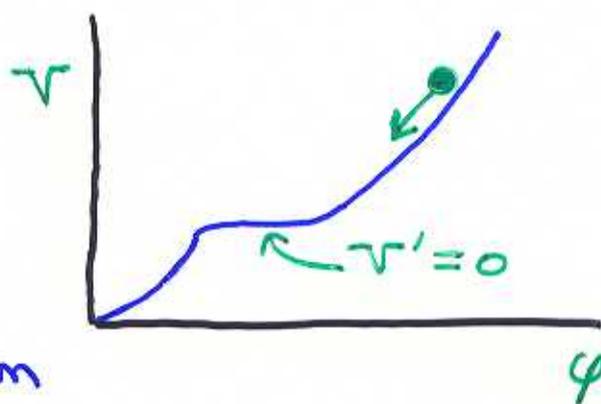
$\Rightarrow 2(1+q)H^2 = 8\pi G \dot{\phi}^2$

* Scalar $\Rightarrow \ddot{\phi} + 3H\dot{\phi} + V'(\bar{\phi}) = 0$

③ Interval of $V'(\phi) = 0$

* $\dot{\phi}(t) = \left[\frac{q_1}{q(t)} \right]^3 \dot{\phi}_1$

* $\frac{1}{2} \dot{\phi}^2 \ll \bar{V} \Rightarrow \Delta \bar{\phi} \approx \frac{\dot{\phi}_1}{3H_1}$



④ Sachs-Wolfe Power Spectrum

* $P_{SW}(k) = \frac{2}{\pi} G k^3 \|U_c(t_{dec}, k)\|^2$ (NB observable)

* Grishchuk's Paradox (PR D50 (1994) 7154)

$P_{SW}(k) \Big|_{slow} = 144 \frac{V_x^3}{G^3 V_x^{1/2}} \rightarrow \infty!!$

* Tsamis & Woodard (astro-ph/0307463) \Rightarrow solved for U_c

$P_{SW}(k) = \frac{9}{4\pi} \frac{G H_x^2}{1+q_x} \|C_x\|^2 \|C_i\|^2 < \infty$

⑤ Back-reaction can be strong!

* And from KE!

* But why this V ???

Allure of Λ -Driven Inflation

① Other Questions

- * why is m_ϕ light?
 - * why initial $\phi(t, \vec{x})$ homogeneous over $\Delta x \gtrsim \frac{1}{aH}$?
- cf Vachaspati & Trodden, gr-qc/9811037

② Λ is perfectly homogeneous!

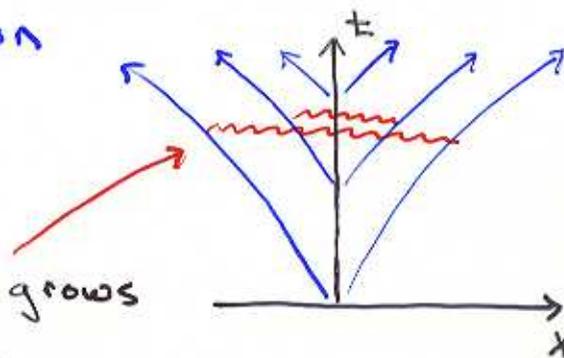
$$\Rightarrow \mathcal{L} = \frac{1}{16\pi G} (R - 2\Lambda) \sqrt{-g} + \mathcal{L}_{\text{matter}}$$

← but not fine tuned

- * gravitons stay massless
- * no problem starting inflation ... but how to stop?

③ Mechanism of QG back-reaction

- * $m=0$ + inflation \Rightarrow particle prod.
- * and no conf. inv. \Rightarrow lots of it!
- * no growth of graviton KE ... but ϕ grows
- * gravity is attractive $\Rightarrow \rho < 0$
- * gravity is weak $\Rightarrow p \approx -\rho$



} \Rightarrow slows inflation

④ Perturbative Results (Tsamis & Woodard, hep-ph/9602315)

- * one loop $\bigcirc + \downarrow = 0$
 - * two loops $\ominus + \oplus + \dots \neq 0$
- $$\Rightarrow \frac{\dot{a}}{a} = H \left\{ 1 - \left(\frac{G\Lambda}{3\pi} \right)^2 \left[\frac{1}{6} (Ht)^2 + \dots \right] + \mathcal{O}(G^3) \right\}$$

$$\langle g_{\mu\nu} dx^\mu dx^\nu \rangle$$

$$= -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x}$$

- * eventually gets non-perti strong ... but then what?

⑤ Advantages (if it works!)

- * no old Λ problem
- * IR effect \Rightarrow use plain, vanilla QG
- * only parameter is $\Lambda \Rightarrow$ predictive

① $Ht = \ln(a)$

② Two sources

(a) Undifferentiated propagators

* $i\Delta(x;x') = \left(\frac{H}{2\pi}\right)^2 \left\{ \frac{1}{y+i\epsilon} - \frac{1}{2} \ln\left(\frac{y+i\epsilon}{aa'}\right) \right\}$

* $y(x;x') \equiv 4 \sin^2\left[\frac{1}{2} H \ell(x;x')\right]$

(b) Integrating over past light-cone

* $\int_{t' > 0} d^4x' \sqrt{-g(x')} \Theta[\text{PLC}(x^\mu)] = \frac{4}{3} \pi H^{-4} \left\{ \ln(a) + \mathcal{O}(1) \right\}$

* integrating $i\Delta$'s gives both!

③ Eg $\langle \frac{\lambda}{4!} \phi^4(x) \rangle$ in $\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\lambda}{4!} \phi^4 + \text{c-term}$

* 2 loops  $= \frac{\lambda}{8} [i\Delta(x;x)]^2 \rightarrow + \frac{\lambda}{8} \left(\frac{H}{2\pi}\right)^4 [\ln(a)]^2$

* 3 loops  $= -\frac{i\lambda^2}{24} \int d^4x' \sqrt{-g(x')} [i\Delta(x;x')]^4 \rightarrow -\frac{\lambda^2}{72} \left(\frac{H}{2\pi}\right)^8 (\ln a)^4$

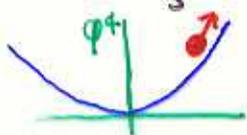
④ Counterterms absorb UV ∞ 's - BUT NOT IR $\ln a$'s - for observables

* Eg $\langle T_{\mu\nu} \rangle$ (Onemli & Woodard, gr-qc/0204065)

$S_{ren} = \lambda \left(\frac{H}{2\pi}\right)^4 \left\{ \frac{1}{8} \ln^2(a) + \frac{1}{18} a^{-3} - \frac{1}{8} \sum_{n=1}^{\infty} \frac{(n+2)}{(n+1)^2} a^{-(n+1)} \right\} + \mathcal{O}(\lambda^2)$

$P_{ren} = \lambda \left(\frac{H}{2\pi}\right)^4 \left\{ -\frac{1}{8} \ln^2(a) - \frac{1}{12} \ln(a) - \frac{1}{24} \sum_{n=1}^{\infty} \frac{n^2-4}{(n+1)^2} a^{-(n+1)} \right\} + \mathcal{O}(\lambda^2)$

* NB $S_{ren} + P_{ren} = \lambda \left(\frac{H}{2\pi}\right)^4 \left\{ 0 - \frac{1}{12} \ln(a) - \frac{1}{6} \sum_{n=1}^{\infty} \frac{(n+2)}{(n+1)} a^{-(n+1)} \right\} + \dots$



physics: inf. prod. grows $\langle \phi^2 \rangle \Rightarrow$ also vac. E.

Stochastic Infl. Gets Leading Logs I

① Origin of stochasticity for $\ddot{\varphi}_\ell + 3H\dot{\varphi}_\ell - \frac{\nabla^2}{a^2}\varphi_\ell = 0$

$$\Rightarrow \varphi_\ell(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left\{ U_A(t, k) e^{i\vec{k}\cdot\vec{x}} \alpha(\vec{k}) + \text{c.c.} \right\}$$

$$* \ddot{U}_A + 3H\dot{U}_A + \frac{k^2}{a^2} U_A = 0 \Rightarrow U_A(t, k) = \frac{H}{\sqrt{2k^3}} \left[1 - \frac{ik}{aH} \right] e^{\frac{ik}{aH}}$$

$$* [\alpha(\vec{k}), \alpha^\dagger(\vec{k}')] = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') \Rightarrow [\tilde{\varphi}_\ell(t, \vec{k}), \tilde{\varphi}_\ell(t, \vec{k}')] = \frac{i}{a^3} \text{ same}$$

$$* UV (k \gg Ha(t)) \Rightarrow U_A \sim \frac{-i}{\sqrt{2k}} \frac{e^{ik/a}}{a} \Rightarrow \text{small \& QM}$$

$$* IR (k \ll Ha(t)) \Rightarrow U_A \sim \frac{H}{\sqrt{2k^3}} \Rightarrow \text{much bigger but CM}$$

$\therefore \tilde{\varphi}_\ell(t, \vec{k})|_{IR}$ is random but classical \Rightarrow Stochastic

② $\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2}{a^2}\varphi + \frac{\lambda}{6}\varphi^3 = 0$ (Starobinsky & Yokoyama, astro-ph/9407016)

$$* \varphi(t, \vec{x}) = \bar{\varphi}(t, \vec{x}) + \Delta\varphi(t, \vec{x})$$

non-linear IR's \uparrow

linear UV's \leftarrow

$$* \Delta\varphi(t, \vec{x}) \equiv \int \frac{d^3k}{(2\pi)^3} \Theta(k - \epsilon Ha(t)) \tilde{\varphi}_\ell(t, \vec{k})$$

Approximations

(a) $|\frac{\Delta\varphi}{\varphi}| \ll 1 \Rightarrow$ neglect $\Delta\varphi^2, \Delta\varphi^3$ & $\Delta\varphi^4$

(b) $\dot{U}_A \sim (\frac{k}{aH})^2 U_A \Rightarrow \Delta\ddot{\varphi} + 3H\Delta\dot{\varphi} - \frac{\nabla^2}{a^2}\Delta\varphi \cong -3Hf(t, \vec{x})$

where $f(t, \vec{x}) \equiv \epsilon H^2 a(t) \int \frac{d^3k}{(2\pi)^3} \delta(k - \epsilon Ha) \tilde{\varphi}_\ell(t, \vec{k})$

(c) $\bar{\varphi}$ is IR $\Rightarrow \ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} - \frac{\nabla^2}{a^2}\bar{\varphi} \cong 3H\dot{\bar{\varphi}}$

$$\therefore 3H\dot{\bar{\varphi}} - 3Hf + \frac{\lambda}{6}\bar{\varphi}^3 \cong 0 \Rightarrow \dot{\bar{\varphi}} \cong -\frac{1}{3H} \frac{\lambda}{6}\bar{\varphi}^3 + f$$

* NB $\langle f(t, \vec{x}) f(t', \vec{x}') \rangle = \frac{H^3}{4\pi^2} \delta(t - t')$

Stoch. Infl. Gets Leading Logs II

③ A known problem \Rightarrow Langevin Eqn

* "White noise" $f(t, \vec{x}) \Rightarrow \langle f(t, \vec{x}) f(t', \vec{x}') \rangle = \frac{H^3}{4\pi^2} \delta(t-t')$

* Random field $\bar{\varphi}(t, \vec{x}) \Rightarrow \dot{\bar{\varphi}} = -\frac{\lambda}{18H} \bar{\varphi}^3 + f$

$$\therefore \langle F(\bar{\varphi}(t, \vec{x})) \rangle = \int d\varphi \rho(t, \varphi) F(\varphi)$$

where $\frac{\partial \rho}{\partial t} = +\frac{\lambda}{18H} \frac{\partial}{\partial \varphi} (\varphi^3 \rho) + \frac{H^3}{8\pi^2} \frac{\partial^2 \rho}{\partial \varphi^2}$

④ Starobinsky's perturbative solution

* $\frac{\partial}{\partial t} \langle \bar{\varphi}^2 \rangle = \int d\varphi \frac{\partial \rho}{\partial t} \varphi^2 = -\frac{\lambda}{9H} \langle \bar{\varphi}^4 \rangle + \frac{H^3}{4\pi^2}$

* $\frac{\partial}{\partial t} \langle \bar{\varphi}^4 \rangle = \int d\varphi \frac{\partial \rho}{\partial t} \varphi^4 = -\frac{2\lambda}{9H} \langle \bar{\varphi}^6 \rangle + \frac{3H^3}{2\pi^2} \langle \bar{\varphi}^2 \rangle$

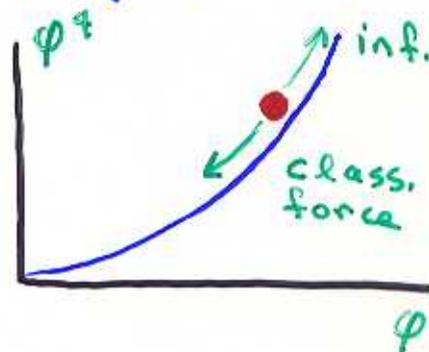
etc. $\Rightarrow \langle \bar{\varphi}^2 \rangle = \left(\frac{H}{2\pi}\right)^2 Ht - \frac{\lambda}{9H^2} \left(\frac{H}{2\pi}\right)^4 (Ht)^3 + \dots$

$$\langle \bar{\varphi}^4 \rangle = 3 \left(\frac{H}{2\pi}\right)^4 (Ht)^2 - \frac{\lambda}{H^2} \left(\frac{H}{2\pi}\right)^6 (Ht)^4 + \dots$$

⑤ Non-pert. solution (Starobinsky & Yokoyama 1994)

* $\lim_{t \rightarrow \infty} \rho(t, \varphi) = N \exp \left[-\frac{\pi^2}{9} \lambda \left(\frac{\varphi}{H}\right)^4 \right]$

* physics: balance of forces



⑥ Conclusions

(a) QFT & Stochastic inflation agree!

(b) Stochastic Infl. sums leading logs (in some cases) by subsuming non-pert. phenomena into classical field eqns!

① The original Eqns

* $\mathcal{L} = \frac{1}{16\pi G} (R - 2\Lambda) \sqrt{-g}$ with $H_{\pm} \equiv \sqrt{\frac{1}{3}\Lambda}$

* $ds^2 = -dt^2 + a^2(t) \overset{g_{ij}(t, \vec{x}) dx^i dx^j}{\cancel{dx^i dx^j}}$ with $H(\pm) \equiv \dot{a}/a$

* $g_{00} \Rightarrow \frac{(3)R}{2a^2} + \frac{1}{8} g^{ij} g^{kl} (g_{ij} g_{kl} - g_{ik} g_{jl}) + H g^{ij} g_{ij} + 3H^2 - 3H_{\pm}^2 = 0$

* $g_{0i} \Rightarrow \frac{1}{2} g^{jk} (g_{ij} g_{jk} - g_{ik} g_{ji}) = 0$

* $g_{ij} \Rightarrow \overset{(3)}{G}_{ij} + \frac{1}{2} a^2 [\delta_i^k \delta_j^l - g_{ij} g^{kl}] (\ddot{g}_{kl} + 3H \dot{g}_{kl}) + a^2 [(-1+2q)H^2 + 3H_{\pm}^2] g_{ij}$
 $+ \frac{a^2}{4} [\delta_i^k \delta_j^l - \frac{1}{2} g_{ij} g^{kl}] g^{mn} \dot{g}_{kl} \dot{g}_{mn} - \frac{a^2}{2} [\delta_i^k \delta_j^l - \frac{3}{4} g_{ij} g^{kl}] g^{mn} \dot{g}_{km} \dot{g}_{ln}$

② What they do perturbatively

* $g_{ij}(t, \vec{x}) \equiv \delta_{ij} + h_{ij}^{\pm\pm} + h^T_{ij} + h^T_{ji} - \frac{1}{2} (\delta_{ij} - \frac{3\partial_i \partial_j}{\nabla^2}) h^L$

* NB $h(t, \vec{x})$ has no $\vec{k}=0$ mode! $+ \frac{1}{2} (\delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2}) h$

* g_{00} eqn $\Rightarrow \frac{1}{2a^2} \nabla^2 (h^L - h) + H \dot{h} + \mathcal{O}(h^2) = 0$

* g_{0i} eqn $\Rightarrow \frac{1}{2} \nabla^2 h^T_i + \frac{1}{2} \partial_i (\dot{h}^L - \dot{h}) + \mathcal{O}(h^2) = 0$

* g_{ij} eqn $\Rightarrow \frac{1}{2} a^2 [\partial_t^2 + 3H \partial_t - \frac{\nabla^2}{a^2}] h_{ij}^{\pm\pm} + \text{trace \& gradient terms} = 0$

$\therefore h_{ij}^{\pm\pm}(t, \vec{x}) = \sqrt{32\pi G} \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda} \left\{ U_{\lambda}(t, \vec{k}) e^{i\vec{k} \cdot \vec{x}} \epsilon_{ij}(\vec{k}, \lambda) \alpha(\vec{k}, \lambda) + \text{c.c.} \right\}$
 is "stochastic source" at $\mathcal{O}(h^2)$ in :

* g_{0i} eqn $\Rightarrow h^T_i$ & $h^L - h$

* g_{00} eqn $\Rightarrow h$

Approximations

① No buildup of IR gravitons (unlike ϕ !!)

$$* h_{ij}^{\pm\pm}(t, \vec{x}) = \sqrt{32\pi G} \int_{\text{IR}} \frac{d^3k}{(2\pi)^3} \sum_{\lambda} \left\{ U_A(t, k) e^{i\vec{k}\cdot\vec{x}} \epsilon_{ij}(\vec{k}, \lambda) \alpha(\vec{k}, \lambda) + \text{c.c.} \right\}$$

* do correct $U_A(t, k)$ as a(t) back-reacts

$$* \ddot{U}_A + 3H\dot{U}_A + \frac{k^2}{a^2} U_A = 0 \quad \text{Exact Soln (Tsamis \& RPW astro-ph/0206010)}$$

② Neglect $\dot{h}_{ij}^{\pm\pm}$ wrt $\frac{\nabla^2}{a^2} h_{ij}^{\pm\pm}$

$$* \text{IR } (k \ll Ha) \Rightarrow U_A(t, k) = \text{const.} + \mathcal{O}\left(\frac{k^2}{H^2 a^2}\right)$$

$$\therefore \dot{U}_A = 0 + \mathcal{O}\left(\frac{k^2}{H^2 a^2}\right) \ll \mathcal{O}\left(\frac{k}{a}\right)$$

* but goi eqn sourced by $h_{ij}^{\pm\pm} \Rightarrow h^T_i \cong 0 \cong h^L_h$

$$\therefore g_{ij}(t, \vec{x}) \cong \delta_{ij} + h_{ij}^{\pm\pm}(t, \vec{x}) + \frac{\partial_i \partial_j}{\nabla^2} h(t, \vec{x})$$

stays small

grows

③ Neglect $\mathcal{O}(h^{\pm\pm, 3})$ & take $\frac{\partial_i \partial_j}{\nabla^2} h \mapsto \frac{1}{3} \delta_{ij} h$

$$* \text{mostly } g_{ij} = \delta_{ij} \left[1 + \frac{1}{3} h\right] \Rightarrow g^{ij} = \frac{\delta^{ij}}{1 + \frac{1}{3} h}$$

$$* \text{goo eqn } 3H^2 \cong 3H_I^2 - \frac{H\dot{h}}{1 + \frac{1}{3}h} \bullet \frac{a^{-2}}{[1 + \frac{1}{3}h]^3} \left\{ \begin{array}{l} \frac{1}{2} h_{ij}^{\pm\pm} \nabla^2 h_{ij}^{\pm\pm} \\ + \frac{3}{8} h_{ij,k}^{\pm\pm} h_{ij,k}^{\pm\pm} \\ - \frac{1}{4} h_{ij,k}^{\pm\pm} h_{kji}^{\pm\pm} \end{array} \right\}$$

④ Neglect $\partial h^{\pm\pm} \partial h^{\pm\pm}$ wrt $h^{\pm\pm} \nabla^2 h^{\pm\pm}$

$$\therefore 3H^2 \cong 3H_I^2 - \frac{H\dot{h}}{1 + \frac{1}{3}h} - \frac{\frac{1}{2} a^{-2} h_{ij}^{\pm\pm} \nabla^2 h_{ij}^{\pm\pm}}{[1 + \frac{1}{3}h]^3}$$

* $\vec{k} \neq 0$ mode gives $h(t, \vec{x})$

* $\int d^3x$ gives $a(t)$

* NB this agrees with Tsamis & Woodard 1996!

$$* h(t, \vec{x}) \approx -\frac{1}{2} \int_0^t dt' \frac{h_{ij}^{tt}(t', \vec{x}) \nabla^2 h_{ij}^{tt}(t', \vec{x})}{H(t') a^2(t')}$$

$$* 3H^2(t) \approx 3H_H^2 - \left\langle \frac{H \dot{h}}{1 + \frac{1}{3}h} - \frac{H \dot{h}}{[1 + \frac{1}{3}h]^3} \right\rangle$$

$$= 3H_H^2 - 3H \frac{\partial}{\partial t} \left\langle \ln \left[1 + \frac{1}{3}h \right] + \frac{\frac{1}{2}}{[1 + \frac{1}{3}h]^2} \right\rangle$$

Borel Summation

$$* F(x) \equiv \sum_{n=0}^{\infty} \frac{F^{(n)}(0)}{n!} x^n$$

$$* \left\{ \begin{array}{l} \langle x^{2n+1} \rangle = 0 \\ \langle x^{2n} \rangle = (2n-1)!! \langle x^2 \rangle^n \end{array} \right\} \Rightarrow \langle F(x) \rangle = \sum_{n=0}^{\infty} \frac{F^{(2n)}}{2^n n!} \langle x^2 \rangle^n (2n-1)!!$$

$$* (2n-1)!! = \prod_{i=1}^n 2i-1 = \int_0^{\infty} dt t^{n-1} e^{-t}$$

$$\Rightarrow \langle F(x) \rangle = \int_0^{\infty} \frac{dt}{\sqrt{2\pi t}} e^{-t} \frac{1}{2} \left\{ F(\sqrt{2t \langle x^2 \rangle}) + F(-\sqrt{2t \langle x^2 \rangle}) \right\}$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} dy e^{-\frac{1}{2}y^2} F(y \sqrt{\langle x^2 \rangle})$$

$$* H^2(t) \approx H_H^2 - H \frac{\partial}{\partial t} \int_0^{\infty} \frac{dy}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left\{ \ln \left[1 + \frac{1}{3}y \sqrt{\langle h^2 \rangle} \right] + \frac{\frac{1}{2}}{[1 + \frac{1}{3}y \sqrt{\langle h^2 \rangle}]^2} \right\}$$

$$\longrightarrow H_H^2 - H(t) \frac{\partial}{\partial t} \int_0^{\infty} \frac{dy}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \left[1 + \frac{3}{2}y^2 - \frac{1}{2}y^4 \right] \ln \left[1 + \frac{y}{3} \sqrt{\langle h^2 \rangle} \right]$$

11/13/03
dS Days

Conclusions

p.9

- ① KE can drive back-reaction
- ② Λ -driven inflation would be ideal
 - if only QG back-reaction can stop it!
- ③ QG does slow inflation perturbatively
 - but we must go non-pert. to prove it
- ④ Stochastic inflation captures the leading logarithms of QFT
 - AND it can sometimes be summed
- ⑤ Have done this for QG
 - still exploring the resulting model